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Fair Artificial Currency Incentives in Repeated Weighted Congestion Games: Equity vs. Equality

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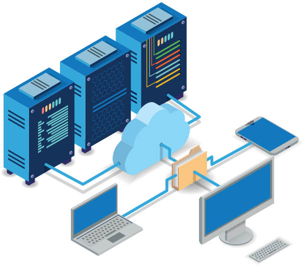
Introduction: The Rise of Sharing Economies



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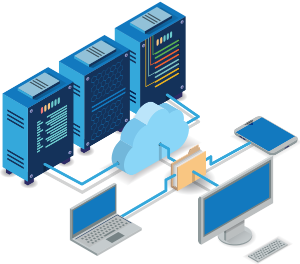

Design allocation **rules/incentives** for **maximum resource utility**

Introduction: Setting

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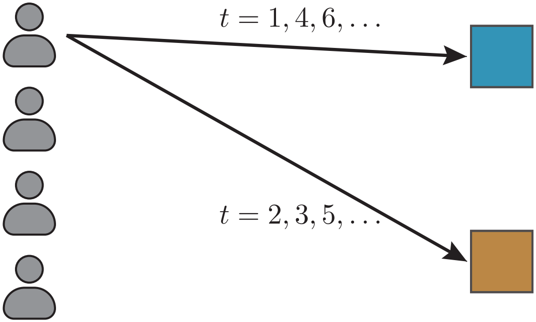

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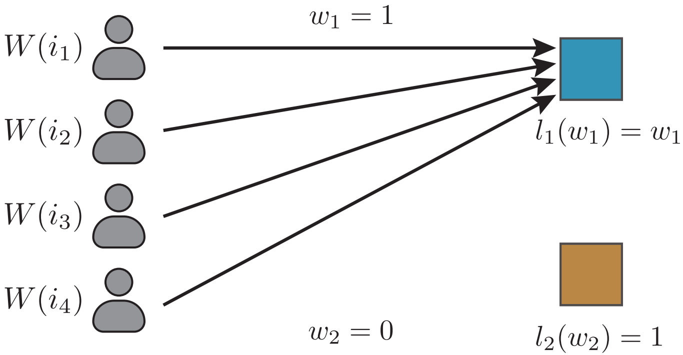

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$$\textbf{No incentives}: C(w) = 1 \cdot 1 + 0 \cdot 1 = 1$$

$$\text{Optimum}: C(w) = 0.5 \cdot 0.5 + 0.5 \cdot 1 = 3/4$$

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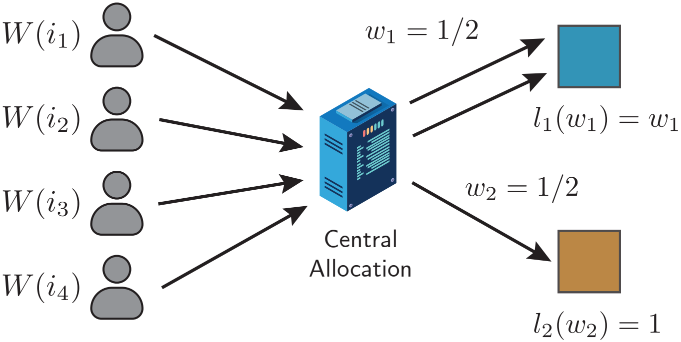

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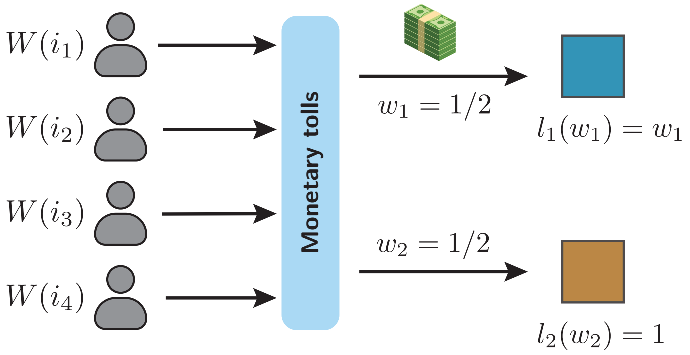

Monetary Tolls

- ✓ Easy to implement
- ✗ Unfair

$$
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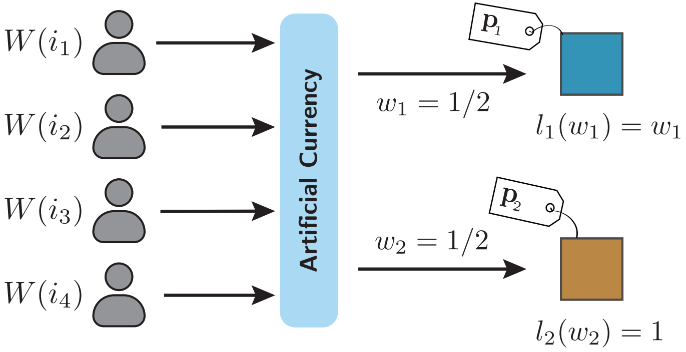

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Artificial Currency

- ✓ AC Payments
- ✓ Fair
- ✓ Turn-taking

Introduction: State-of-the-art

Design AC incentives that are **societally-optimal** and **maximize fairness**

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Artificial Currency Incentives:

- Bidding (Censi et al., 2019), (Elokda et al., 2023)
- **Fixed-prices** (Salazar et al., 2021), (Pedroso et al., 2023)

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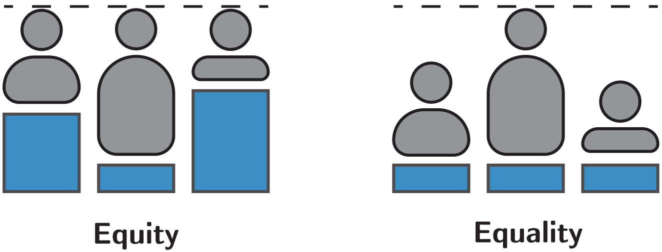
Artificial Currency Incentives:

- Bidding (Censi et al., 2019), (Elokda et al., 2023)
- **Fixed-prices** (Salazar et al., 2021), (Pedroso et al., 2023)

Missing: Formal **definition** of **fairness metrics**
Missing: AC **design** maximizing **fairness metrics**

Introduction: Equity vs. Equality



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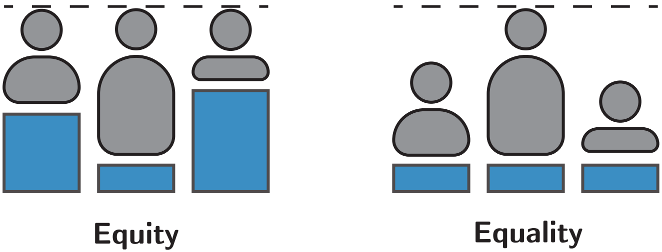
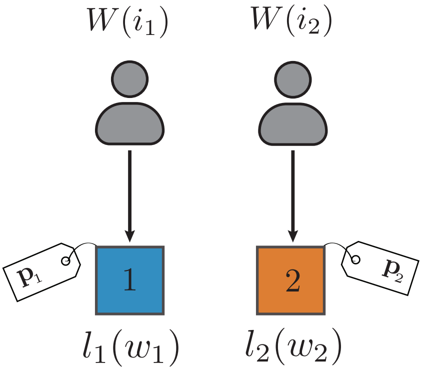


Design for **equity** vs. Design for **equality**

Problem Statement: Setting

- Players: $i \in \Omega = [0, 1]$
- Resources: $r \in \mathcal{R} = \{1, 2\}$
- Participation probability: P_{go}



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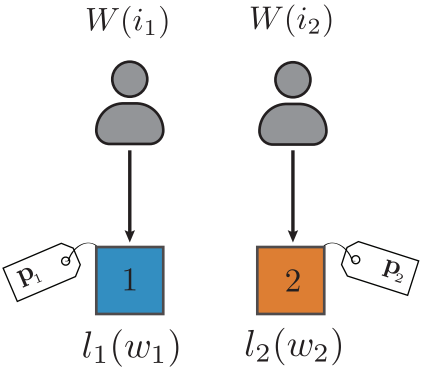
- AC level at time t: $K_t(i) \geq 0$
- Resource prices: $\mathbf{p}_1(w), \mathbf{p}_2(w)$

$$K_{t+1}(i) = \begin{cases} K_t(i) - \mathbf{p}_1(W(i)), & \text{chooses 1} \\ K_t(i) - \mathbf{p}_2(W(i)), & \text{chooses 2} \\ K_t(i), & \text{no participation at t} \end{cases}$$



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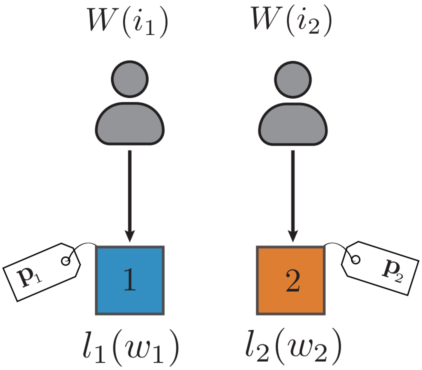

Most uncomfortable resource has **negative price**

Problem Statement: Player Decision Model

Making a **decision** at time t a player i ponders:

- The **perceived discomfort** at time t: $U_t(i) l_r(w_r)$
- **Future decision constraints** due to future AC level

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Augmented cost :

$$c^r(i) = \min_{\bar{\mathbf{y}} \in \mathbb{R}^2_{\geq 0}} U_t(i) l_r(w_r) + \mathbb{E}[U_t] P_{\text{go}} T \bar{\mathbf{y}}^\top \mathbf{l}(w)$$

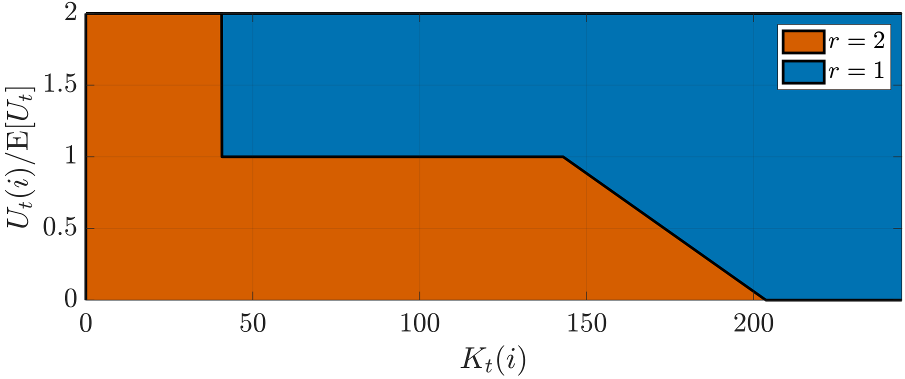
$$\text{s.t. } \mathbf{1}^\top \bar{\mathbf{y}} = 1$$

$$K_t(i) - \mathbf{p}_r(W(i)) - P_{\text{go}} T \bar{\mathbf{y}}^\top \mathbf{p}(W(i)) \geq 0$$

Decision :

$$A_t(i) \in \operatorname*{argmin}_{r \in \mathcal{R}} c^r(i)$$

Problem Statement: Player Decision Model



Problem Statement: Efficiency

Definition (Nash Equilibrium)

$A_t : \Omega \to \{0, 1, 2\}$ is a NE if $\forall i \; \forall a$

$$c^r_{\mathbf{w}^{A_t}}(i) \leq c^a_{\mathbf{w}^{A_t}}(i)$$

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Societal Cost: $C(\mathbf{w}^{A_t})$

Definition (Price of Anarchy)

$$\mathrm{PoA}_t := \frac{\max_{A_t \in \mathcal{A}_t^{NE}} C(\mathbf{w}^{A_t})}{\min_{A_t} C(\mathbf{w}^{A_t})} = \frac{\text{worst NE equilibrium}}{\text{societal optimum}} \quad \text{(1 at societal optimum)}$$

Problem Statement: Fairness

Average endured latency of player i until t:

$$L_t(i) = \underbrace{\frac{1}{N_t(i)}}_{\substack{\text{Number of times } i \\ \text{participated until } t}} \sum_{\substack{\tau=0 \\ A_\tau(i)\neq 0}}^{t} \underbrace{l_{A_\tau(i)}(\mathbf{w}^{A_\tau})}_{\substack{\text{latency of } i \\ \text{at time } \tau}}$$

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Definition (Equity and Equality)

$$\mathrm{InEqt}_t^2 := \mathrm{Var}[L_t] \quad \text{(ideally 0)}$$

$$\mathrm{InEql}_t^2 := \mathrm{Var}[L_t/W] \quad \text{(ideally 0)}$$

Problem Statement: Design Problem

Problem (AC Incentive Design Problem)

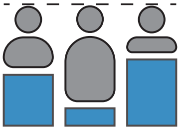
Design $\mathbf{p}_1(w)$, $\mathbf{p}_2(w)$ such that

- $\mathrm{PoA}_t \rightarrow 1$
- $\mathrm{InEqt}_t \rightarrow 0$ or $\mathrm{InEql}_t \rightarrow 0$

Incentive Design: Equity

Equity: All players endure the same latency on average **irrespective of their weight**

$\implies$ **weight-independent** prices



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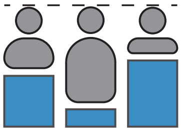
Efficiency: Global **AC level constant at SO**, i.e., $\lim_{t\to\infty} \mathbb{E}[K_{t+1}] - \mathbb{E}[K_t] = 0$

$$\implies \mathbf{p}^\top \mathbf{w}^\star = 0$$



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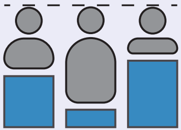
Theorem (Design for Equity)

For all $\epsilon > 0$, there exists $\delta \propto \epsilon$

$$\mathbf{p}(w) = S[\mathrm{rat}_\delta(\mathbf{w}_2^\star/\mathbf{w}_1^\star)\ -1]^\top$$

- $\mathrm{PoA}_t \to \mathrm{PoA}_\infty \leq 1 + \epsilon$
- $\mathrm{InEqt}_t \to 0$

where $S \in \mathbb{Q}_{>0}$.



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$$\mathbf{p}(w) = \begin{cases} S\left[\mathrm{rat}_\delta\left(\frac{n_2(w,\theta^\star)}{n_1(w,\theta^\star)}\right) \; -1\right]^\top, & \frac{w}{\theta^\star} \leq 1 \\ S\mathrm{rat}_\delta\left(\frac{\mathbf{w}_2^\star}{\mathbf{w}_1^\star}\right)\left[1 \; -\mathrm{rat}_\delta\left(\frac{n_1(w,\theta^\star)}{n_2(w,\theta^\star)}\right)\right]^\top, & \frac{w}{\theta^\star} > 1, \end{cases}$$

- $\mathrm{PoA}_t \to \mathrm{PoA}_\infty \leq 1 + \epsilon$
- $|\mathrm{InEql}_t - \mathrm{InEql}^\star| \to |\mathrm{InEql}_\infty - \mathrm{InEql}^\star| < \delta_1$

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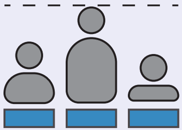

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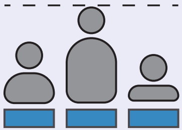
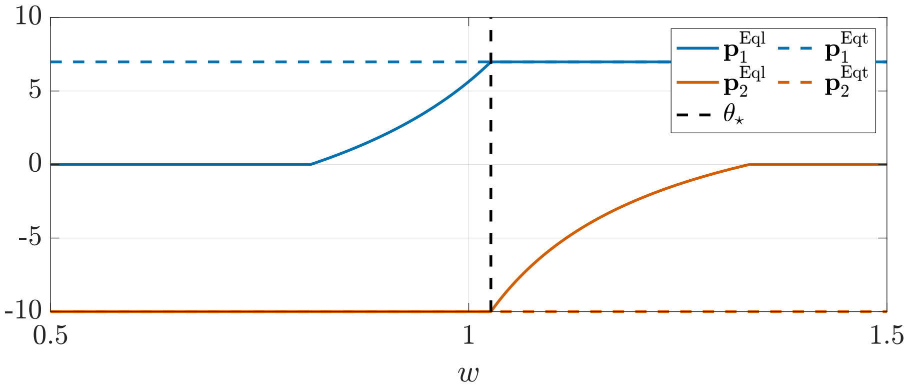
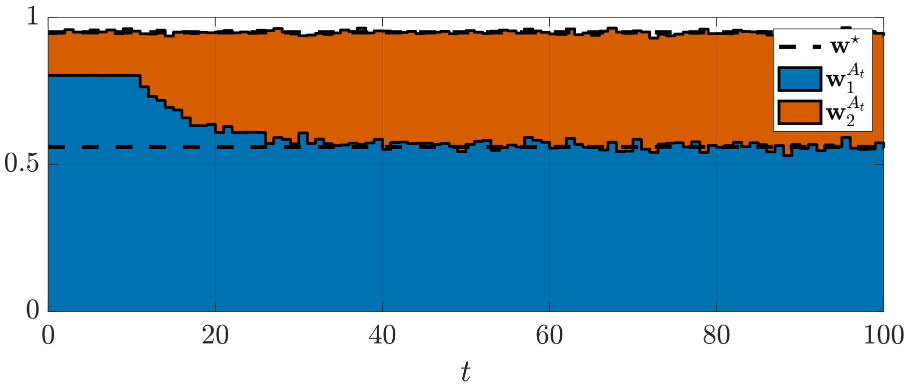


But: It may **not** be possible to achieve perfect equality

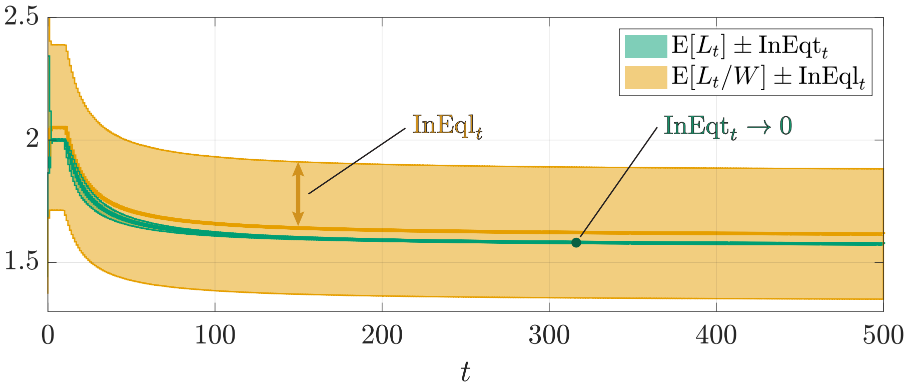
Results: Incentive Design



Results: Aggregate decision



Results: Design for Equity



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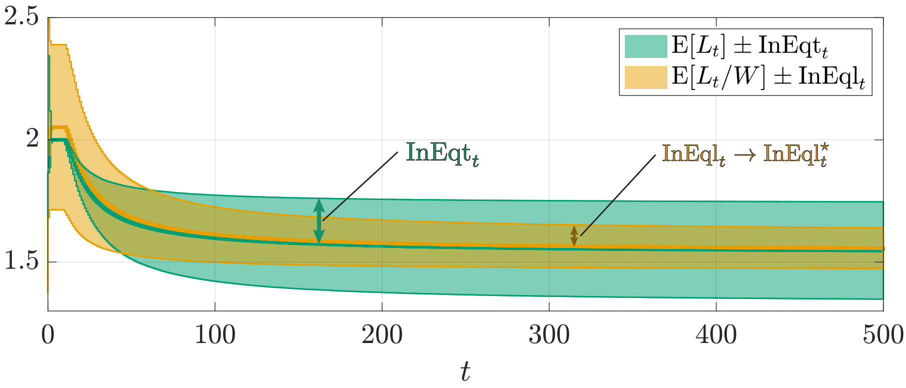
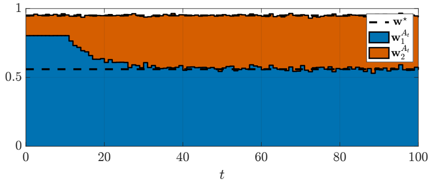
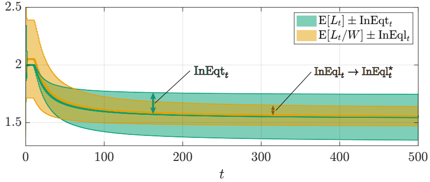


Conclusion





- **Fair** AC incentive scheme
- Formal definition of **equality** and **equity**
- AC **incentive design** for equity/equality
- **Societal-optimum** is achieved
- Always possible to achieve **perfect equity**
- May be impossible to achieve **perfect equality**

http://fish-tue.github.io