62nd IEEE Conference on Decision and Control

Urgency-aware Routing in Single Origin-destination Itineraries through Artificial Currencies

Leonardo Pedroso[1] W.P.M.H. (Maurice) Heemels[1] Mauro Salazar[1]

[1]Control Systems Technology section,
Eindhoven University of Technology,
The Netherlands

Introduction

Motivation

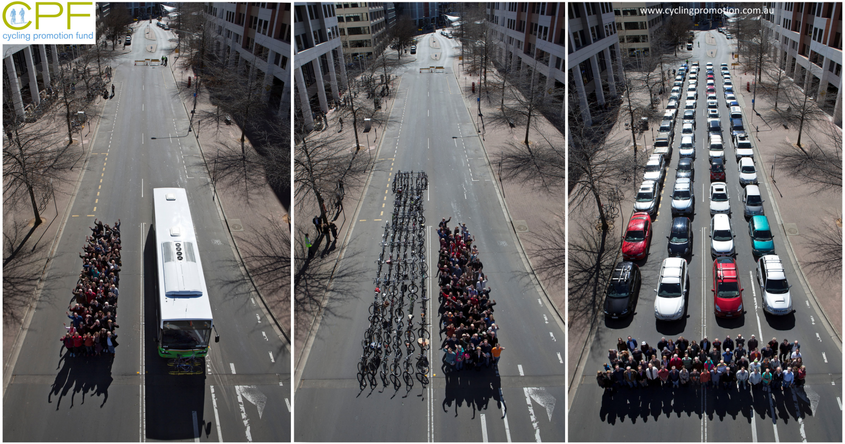


Figure 1: 69 people in bus, bikes, and cars. (Cycling Promotion Fund, 9th September 2012 [C.P.F., 2012])

Introduction

Opportunity

- Vehicle autonomy
- Car sharing
- Public transport
- Connectivity

Figure 2: New opportunities. [Raysonho CC0,Dullu CC BY-SA 4.0]

Centralized controlled **intermodal mobility** → **system's optimum** performance![1,2]

[1] Salazar, Rossi, Schiffer, Onder, Pavone. "On the interaction between autonomous mobility-on-demand and public transportation systems." ITSC, 2018. [Salazar et al., 2018]

[2] Wollenstein-Betech, Salazar, *et al.*. "Routing and rebalancing intermodal autonomous mobility-on-demand systems in mixed traffic." IEEE T-ITS, 2021. [Wollenstein-Betech et al., 2021]

Literature Review

Self-interested behavior ⟷ **Societal Welfare**

Monetary Tolls [1]

- ✓ Easy to implement
- ✓ Easy to use
- ✗ Unfair

Artificial Currencies [2]

- ✓ Fair
- ✗ Bidding
- ✗ Uncertainty

Idea: Bridge the gap[3]

Payment-transaction of **artificial currency** → **urgency-aware** system's **optimum**

[1] [Pigou, 1920, Morrison, 1986, Bergendorff et al., 1997, Fleischer et al., 2004, Paccagnan et al., 2019]

[2] [Prendergast, 2016, Gorokh et al., 2019, Censi et al., 2019, Elokda et al., 2022]

Problem Statement

Repeated game-framework

- User choice: $\mathbf{y}^i(t) \in \{0,1\}^n$
- Traveling probability: P_{go}
- Each arc has a **price**:

$$k^i(t+1) = k^i(t) - \mathbf{p}^\top \mathbf{y}^i(t)$$

- **Aggregate** flows of M users:

$$\mathbf{x}(t) = \frac{1}{M}\sum_i \mathbf{y}^i(t)$$

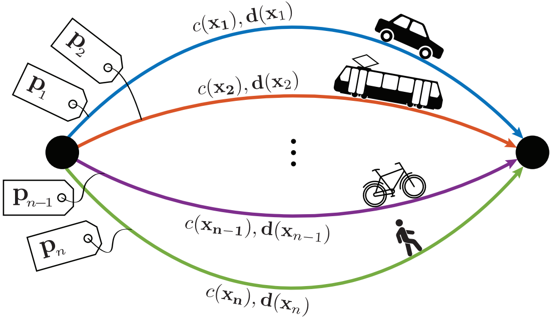


Figure 3: Parallel-arc network.

Self-interested at a cost ⚔ **Altruistic** for a reward

Problem Statement

Three-level Analysis

Three-level Analysis

Macroscopic
(Central Operator)

Microscopic
(User)

Mesoscopic
(Incentive Mechanism)

Problem Statement

Three-level Analysis: Macroscopic

- **Social** cost of arc j: $\mathbf{c}_j(\mathbf{x}_j)$
- Minimize overall social cost: $\mathbf{c}^\top \mathbf{x}$

Problem (Central Operator's Problem)

The **central operator** aims at routing customers so that the **aggregate flows** are

$$\mathbf{x}^\star \in \arg \min_{\mathbf{x}\in[0,1]^n} \mathbf{c}(\mathbf{x})^\top \mathbf{x}$$
$$\text{s.t.} \qquad \mathbf{1}^\top \mathbf{x} = P_{\text{go}}.$$

Problem Statement

Three-level Analysis: Microscopic

- **Discomfort** of arc j: $\mathbf{d}_j(\mathbf{x}_j)$
- Daily **sensitivity** to discomfort: s^i
- Min. **daily perceived** discomfort + average **future** discomfort over T days

Problem (Individual User's Problem)

A traveling user with Karma level $k \geq 0$, reference k_{ref}, and sensitivity s will choose his/her route as $\mathbf{y}^\star$ resulting from

$$(\mathbf{y}^\star, \bar{\mathbf{y}}^\star) \in \underset{\mathbf{y}\in\mathcal{Y},\, \bar{\mathbf{y}}\in\bar{\mathcal{Y}}}{\operatorname{argmin}} \; s\, \mathbf{d}(\mathbf{x})^\top \mathbf{y} + T\, \bar{s}\, \mathbf{d}(\mathbf{x})^\top \bar{\mathbf{y}}$$
$$\text{s.t.} \;\; k - \mathbf{p}^\top \mathbf{y} - T\mathbf{p}^\top \bar{\mathbf{y}} \geq 0$$
$$\mathbf{p}^\top \mathbf{y} \leq k,$$

Problem Statement

Three-level Analysis: Mesoscopic

- **Infinite-user** population: $M \to \infty$
- Users achieve daily **Wardrop Equilibrium** (WE): $\mathbf{x}^{\mathrm{WE}}(t)$
- Design **prices p**

Problem (Pricing Problem)

Given a desired system optimum $\mathbf{x}^{\star}$, select $\mathbf{p} \in \mathbb{R}^n$ so that

$$\lim_{t \to \infty} \mathbf{x}^{\mathrm{WE}}(t) = \mathbf{x}^{\star}.$$

Best-response strategy

Closed-form Solution

Theorem (User's Best Response Strategy)

An **optimal response strategy** of a with Karma k, sensitivity s, and Karma reference k_{ref} is $\mathbf{y}^\star = \mathbf{e}_{\mathbf{j}^\star}$ iff

$$\bar{\gamma}_{j^\star} \geq \underline{\gamma}_{j^\star} \quad \text{and} \quad \gamma_{j^\star} \leq s/\bar{s} \leq \gamma_{j^\star-1}$$

Best-response strategy

Closed-form Solution

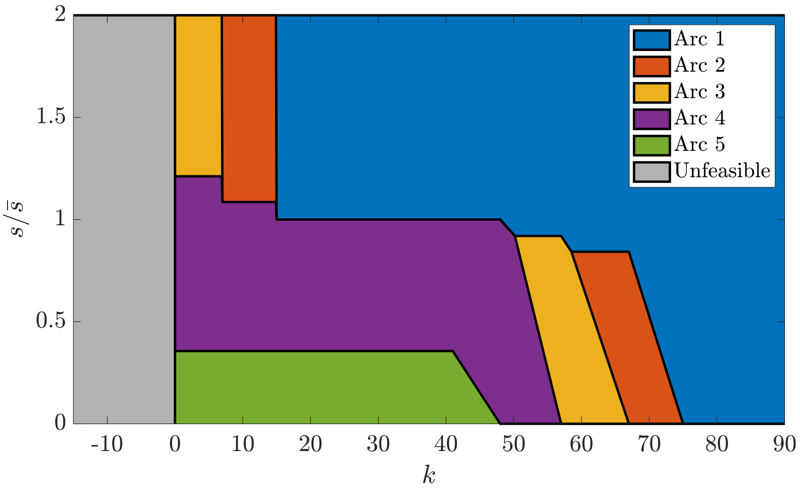


Figure 4: Decision landscape of individual user's problem.

Pricing Design Problem

At **steady-state** in $\mathbf{x}^\star$

- **Total Karma** remains constant: $\mathbf{p}^\top \mathbf{x}^\star = 0$

For $n = 2$ arcs[1]

- $\mathbf{p}^\top \mathbf{x}^\star = 0$ **alone** defines the **optimal prices**

For n arcs

- Much more intricate

[1] Salazar, Paccagnan, Agazzi, Heemels. "Urgency-aware optimal routing in repeated games through artificial currencies." European Journal of Control 62 (2021). [Salazar et al., 2021]

Pricing Design Problem: n arcs

Markov chain

- $\mathrm{P}(j^\star|k^i, \mathbf{p}, \mathbf{x}^\star)$ from the **best response strategy**
- Stationary **Karma distribution** $\pi_\infty(\mathbf{p}, \mathbf{x}^\star)$

Aggregate of Markov chains

$$\mathbf{x}_j^\star = \sum_{k=k_{\min}}^{k_{\max}} \mathrm{P}(j^\star = j|k, \mathbf{p}, \mathbf{x}^\star)[\pi_\infty(\mathbf{p}, \mathbf{x}^\star)]_k, \quad j = 1, \ldots, n$$

Challenge for $n > 2$

- The support of the chain depends on $\mathbf{p}$
- **Gradient-free** optimization

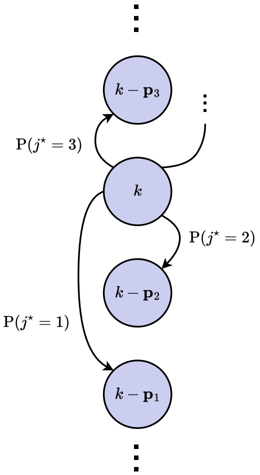


Figure 5: Markov chain.

Numerical Results

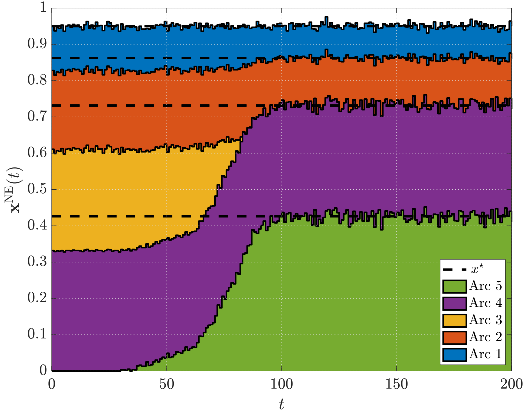


Figure 6: Aggregate flows.

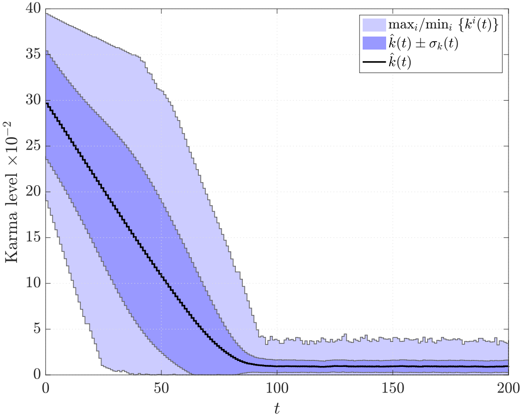


Figure 7: Karma level.

Numerical Results

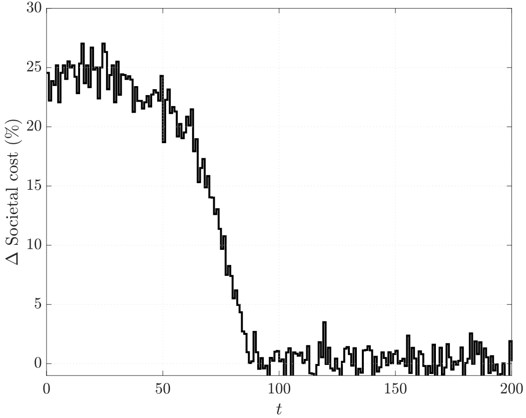


Figure 8: Societal cost.

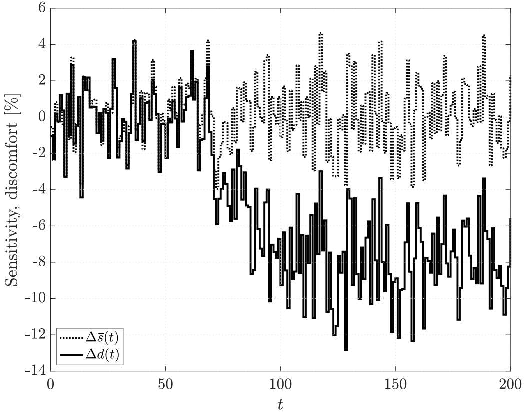
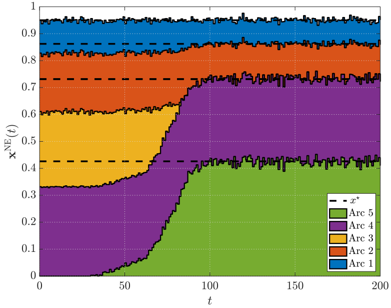


Figure 9: Sensitivity w.r.t. urgency-unaware.

Conclusion



- Incentive scheme: **fair** and **urgency-aware**
- Solution for the **user's best response strategy**
- **Pricing design** procedure for n arcs
- Aggregate decision achieves **system's optimum**
- 8% **improvement** w.r.t. urgency-unaware policy

http://fish-tue.github.io

References I

Bergendorff, P., Hearn, D. W., and Ramana, M. V. (1997).
Congestion toll pricing of traffic networks.
Network Optimization, pages 51–71.

Censi, A., Bolognani, S., Zilly, J. G., Mousavi, S. S., and Frazzoli, E. (2019).
Today me, tomorrow thee: Efficient resource allocation in competitive settings using karma games.
In *Proc. IEEE Int. Conf. on Intelligent Transportation Systems*.

C.P.F. (2012).
Cycling promotion fund media release (`http://www.cyclingpromotion.com.au/content/view/566/9/`).

Dllu.
CC BY-SA 4.0 ¡https://creativecommons.org/licenses/by-sa/4.0¿, via Wikimedia Commons.

Elokda, E., Cenedese, C., Zhang, K., Lygeros, J., and Dörfler, F. (2022).
Carma: Fair and efficient bottleneck congestion management with karma.
arXiv preprint arXiv:2208.07113.

Fleischer, L., Jain, K., and Mahdian, M. (2004).
Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games.
In *IEEE Symp. on Foundations of Computer Science*.

Gorokh, A., Banerjee, S., and Iyer, K. (2019).
From monetary to non-monetary mechanism design via artificial currencies.
Available at SSRN 2964082.

References II

Morrison, S. A. (1986).
A survey of road pricing.
Transportation Research Part A: Policy and Practice, 20(2):87–97.

Paccagnan, D., Chandan, R., Ferguson, B. L., and Marden, J. R. (2019).
Incentivizing efficient use of shared infrastructure: Optimal tolls in congestion games.
arXiv preprint arXiv:1911.09806.

Pigou, A. C. (1920).
The Economics of Welfare.
Macmillan.

Prendergast, C. (2016).
The allocation of food to food banks.
EAI Endorsed Trans. Serious Games, 3(10).

Raysonho.
Open Grid Scheduler / Grid Engine, CC0, via Wikimedia Commons https://commons.wikimedia.org/wiki/File:BikeShareTorontoTD.jpg.

Salazar, M., Paccagnan, D., Agazzi, A., and Heemels, W. M. (2021).
Urgency-aware optimal routing in repeated games through artificial currencies.
European Journal of Control, 62:22–32.

Salazar, M., Rossi, F., Schiffer, M., Onder, C. H., and Pavone, M. (2018).
On the interaction between autonomous mobility-on-demand and public transportation systems.
In *2018 21st international conference on intelligent transportation systems (ITSC)*, pages 2262–2269. IEEE.

References III

Wollenstein-Betech, S., Salazar, M., Houshmand, A., Pavone, M., Paschalidis, I. C., and Cassandras, C. G. (2021).
Routing and rebalancing intermodal autonomous mobility-on-demand systems in mixed traffic.
IEEE Transactions on Intelligent Transportation Systems, 23(8):12263–12275.